Power Allocation for Multiband Coded OFDM Systems with Limited Feedback

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***Abstract*—In this paper, we study the power allocation for multiband coded OFDM systems. With limited feedback, we propose an effective power allocation method across OFDM bands to maximize the throughput and achieve the quality of service target. To facilitate the proposed method, two optimization algorithms based on greedy and dynamic programming principles are discussed. The trade-off between performance and complexity is provided. Simulation results show that the proposed power allocation mechanism allows a signal to noise ratio gain of 2 dB at a goodput of 2.5 bit per second per Hz over the multiband OFDM systems with equal power allocation.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) in conjunction with simple one-tap equalizers is an effective technique to combat frequency selective fading. In OFDM, a high data rate stream is divided into multiple sub-streams of lower data rates, which are transmitted over narrow bandwidth flat fading channels, each is carried by a subcarrier, without interference from a subcarrier to another. As there is a variation of channel gains for different subcarriers, coding over the frequency domain can exploit a frequency diversity. A scheme that employs a channel code followed by a bitwise interleaver is referred to as coded OFDM, which has been widely used in various standards, e.g., IEEE 802.11a [1] and IEEE 802.16 [2].

In a practical OFDM system, the average transmitted power is often limited. As such, according to the water-filling theorem, a higher power level should be allocated to the subcarrier that experiences a better channel gain so that it can carry signals of a higher data rate to maximize the overall throughput [3]. The mechanism of allocating power levels and assigning data rates for different subcarriers is referred to as the adaptive bit-loading technique. Based on this technique, various adaptive modulation and coding (AMC) systems were proposed in [4]–[6]. In these systems, feedback links of high data rate are required to provide the transmitter with all the subcarrier gains. Excessive feedback loads may lead to a difficulty in practical implementation. In order to reduce the amount of feedback information, a more realistic system called OFDM with symbol-level adaptive modulation and coding (symbol-level AMC) was proposed in [7]. In this symbol-level scheme, the signals transmitted by all the subcarriers of an OFDM symbol (which is a codeword) are encoded and modulated by a single pair of encoder and modulation scheme, which is adaptively decided by the receiver based on a prediction of channel state information (CSI). This scheme was further discussed in [8] for channels with relatively short delay spreads. Note that in symbol-level AMC, since no feedback information on each subcarrier is available, adaptive bit-loading is not possible and an optimal performance is not achieved, which is the price of limited feedback.

For an ultra-wideband (UWB) system, which is supported with a very wide bandwidth, signals can be transmitted over multiple OFDM bands. The resulting system is called multiband OFDM UWB which has been defined in IEEE 802.15.3a [9]. Due to the frequency selectivity of the channel, different OFDM bands can experience different channel conditions. Furthermore, the whole bandwidth consisting of multiple OFDM bands can share a fixed amount of power budget. Motivated by these conditions and inspired by the approach in [7], [8], we propose a new and efficient power allocation method over OFDM bands based on the greedy and dynamic programming principles. In this method, the receiver predicts the CSI of all OFDM bands and then accordingly allocates suitable transmission power levels and corresponding modulation and coding schemes[1]. The advantages of the proposed method are as follows: i) Since the receiver only needs to feedback a transmission power level and a pair of modulation format and coding for each OFDM band, the total amount of feedback information can be kept significantly lower than that of a conventional system which provides feedback for all subcarriers; ii) Rather than allocating power for all OFDM symbols equally in the conventional symbol level AMC approach in [7], [8], the proposed method allows us to enjoy more freedom in allocating transmission power over multiple OFDM symbols. Therefore, the average throughput is improved because an OFDM band in a better condition will be assigned with a higher power level and can transmit in a higher data rate effectively. Simulation results will show that the proposed power allocation method achieves a significant SNR gain compared with the system with equal power allocation.

[1]This method could be applied to single band OFDM systems where the power allocation is carried out over groups of OFDM symbols (in the time domain) provided that the CSI of future OFDM symbols are predicted.

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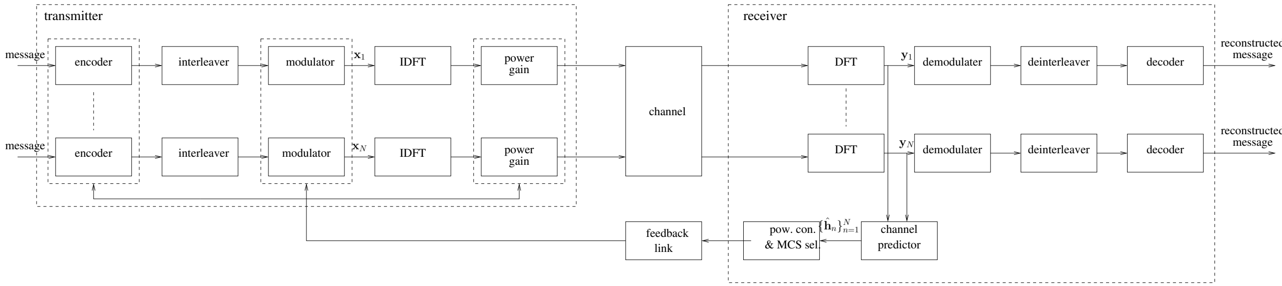

Fig. 1. Block diagram of the proposed system.

The rest of this paper is organized as follows. Sec. II presents the system model and the proposed power allocation method. This method is then mathematically formulated to be an optimization problem in Sec. III. Sec. IV shows simulation results. Finally, Sec. V offers concluding remarks.

II. Power allocation for multiband OFDM systems

A. Multiband OFDM system model

The block diagram of a coded OFDM system is illustrated in Fig. 1. There are N OFDM bands in this system sharing a fixed power budget to transmit OFDM signals. For each band, the transmitter has a chain of a channel encoder, a bitwise interleaver, a modulator, and an inverse discrete Fourier transform (IDFT) operator. Let us consider the signals which are transmitted in the nth OFDM band, where $n = 1, 2, \ldots, N$. After going through a channel encoder, an interleaver, and a symbol mapper/modulator where Gray mapping is used, the transmitted message becomes a symbol sequence of length L, which is denoted by $\mathbf{x}_n = [x_{n,0}\ x_{n,1}\ \ldots\ x_{n,L-1}]^{\mathrm{T}}$. We make the following assumptions: i) the average energy of transmitted symbols is unit; ii) the channel remains unchanged during each OFDM symbol interval but varies from one symbol to another; iii) no ISI is observed due to cyclic prefix (CP); and iv) for simplicity, each OFDM symbol consists of only one codeword, i.e., the number of subcarriers is L.

Let $\mathbf{h}_n = [h_{n,0}\ h_{n,1}\ \ldots\ h_{n,P-1}]^{\mathrm{T}}$ be the channel impulse response (CIR) vector for the considered OFDM symbol transmitted in the nth OFDM band. Note that $\mathbf{h}_n$'s can be different for different bands. The received signal at the lth subcarrier after taking the discrete Fourier transform (DFT) is given by

$$y_{n,l} = \sqrt{E_n} g_{n,l} x_{n,l} + w_{n,l},\ l = 0, 1, \ldots, L-1, \quad (1)$$

where E_n is the transmission power allocated for the nth OFDM band, $g_{n,l}$ is the lth subcarrier's gain, which is obtained by

$$g_{n,l} = \sum_{p=0}^{P-1} h_{n,p} \exp\left(-\frac{j2\pi lp}{L}\right),\ l = 0, 1, \ldots, L-1, \quad (2)$$

and $\{w_{n,l}\}$ is an independent and identically distributed (iid) background noise sequence of circularly symmetric complex Gaussian (CSCG) random variables with the same variance σ^2, i.e., $w_{n,l} \sim \mathcal{CN}(0, \sigma^2)$. For convenience, let $E[\|\mathbf{h}_n\|^2] = 1$ and $\sigma^2 = 1$. The average signal-to-noise ratio (SNR) for the OFDM symbol transmitted in this band, called OFDM symbol SNR (OS-SNR), is therefore given by E_n. At the receiver, the received signal vector $\mathbf{y}_n = [y_{n,0}\ y_{n,1}\ \ldots\ y_{n,L-1}]^{\mathrm{T}}$, is used as the input of a soft-output demodulator. The output of the demodulator is fed into a channel decoder which then reconstructs the original message.

B. Power allocation over multiple OFDM bands

A new power control method for a multiband coded OFDM system is proposed as follows. In the system, there is a set of predefined modulation and coding pairs with each being referred to as a modulation and coding scheme (MCS) candidate. To transmit an OFDM symbol in an OFDM band, the transmitter can select one of the MCS candidates in this set and a suitable power level. Importantly, in our proposed mechanism, the MCS assignment and power allocation are decided by the receiver. In performing a power control policy, the following steps are carried out:

Step 1: The receiver first predicts the CIR vectors for all the N bands.

Step 2: Based on predicted CIR vectors for the given OFDM bands, the receiver then allocates power levels for all the bands so as to maximize the total data rate subject to the constraints that i) the sum of allocated powers does not exceed a predefined power level and ii) a certain bit error rate (BER) target is achieved for the assigned MCS to each band. The predefined power level is called the power budget. The BER target is the parameter to control the quality of service (QoS).

Step 3: Finally, the allocated power levels and assigned MCSs are sent to the transmitter via a feedback link.

In our proposed mechanism, deriving an efficient algorithm in Step 2 plays an essential role and this will be discussed in Section III.

III. Power allocation policy: Problem formulation and algorithms

In this section, we first formulate the optimization problem for our proposed power allocation method and then discuss two low-complexity algorithms based on the greedy and dynamic programming principles.

A. Problem formulation

We assume that the system uses a set $\mathcal{M}$ of M MCS candidates with the data rates $\eta^{(m)}$, $m = 1, 2, \ldots, M$, where $0 = \eta^{(1)} < \eta^{(2)} < \ldots < \eta^{(M)}$. Each MCS candidate is represented by an index in the set $\{1, 2, \ldots, M\}$. In order to perform a power allocation policy, it is assumed that the predicted CIR vectors for all N OFDM bands are available, which are denoted by $\hat{\mathbf{h}}_1, \hat{\mathbf{h}}_2, \ldots, \hat{\mathbf{h}}_N$. Furthermore, for a given CIR vector, it is also assumed that the receiver can estimate the BER performance corresponding to a specific MCS. Let $P_b(E; m, \mathbf{h})$ denote the estimated BER function, where E is the transmission power, m is the index for an assigned MCS candidate, and $\mathbf{h}$ is the CIR vector. Since the data rate corresponding to the MCS index 1 is zero, $P_b(E; 1, \mathbf{h}) = 0,\ \forall E \geq 0$, i.e, if the data rate is zero, there is no error for any transmission power level.

Let E_T and $\bar{P}_b$ be the power budget and the target BER, respectively. Note that we set the same target BER for all OFDM bands. In addition, $\bar{P}_b$ is chosen to be sufficiently low such that the system can achieve a good performance. The problem is to assign MCSs and allocate power levels for transmitting OFDM symbols over N OFDM bands constrained on the conditions that the sum of power levels does not excess E_T and the estimated BERs are not greater than $\bar{P}_b$. In an efficient power allocation policy, we expect that an OFDM band suffering from a worse channel condition is allocated a lower power level or even no power in order to save the power for the other bands which are in better conditions. Note that if no power is allocated for an OFDM symbol, the data rate is zero as the MCS index 1 is chosen. The problem is formulated as follows:

$$\begin{aligned} \max_{\substack{m_1,\ldots,m_N \\ E_1,\ldots,E_N}} & \sum_{n=1}^{N} \eta^{(m_n)}, \qquad (3) \\ \text{s. t. } & E_n \geq 0,\ \forall k : 1 \leq n \leq N, \\ & \sum_{k=1}^{N} E_n \leq E_T, \\ & P_b(E_n; m_n, \hat{\mathbf{h}}_n) \leq \bar{P}_b,\ \forall n : 1 \leq n \leq N, \end{aligned}$$

where E_n and m_n with $n \in \{1, 2, \ldots, N\}$, are the allocated power level and assigned MCS index for the nth OFDM band, respectively. Note that there are $2N$ variables to be controlled in (3). However, because of the QoS constraint, the selection of an MCS index, say m_n, strongly depends on the corresponding power level, i.e, E_n. Assuming that E and $\hat{\mathbf{h}}$ are given for an OFDM band, we cannot choose the MCS index for this band with a value greater than

$$m_n^*(E) = \max_{\substack{m \in \{1,2,\ldots,M\} \\ P_b(E;m,\hat{\mathbf{h}}) \leq \bar{P}_b}} m \qquad (4)$$

since $P_b(E; 1, \mathbf{h}) \leq P_b(E; 2, \mathbf{h}) \leq \ldots \leq P_b(E; M, \mathbf{h})$ (BER always increases as the data rate increases for a fixed SNR (or transmission power)). Therefore, to achieve the maximum total data rate, we should always select $m_n^*(E)$ for OFDM band n if E power is given for this band. From the discussions above and by letting

$$\eta_n^*(E) = \eta^{m_n^*(E)}, \qquad (5)$$

the problem in (3) becomes

$$\begin{aligned} \max_{E_1,\ldots,E_N} & \sum_{n=1}^{N} \eta_n^*(E_n), \qquad (6) \\ \text{s. t. } & E_n \geq 0,\ \forall n : 1 \leq n \leq N, \\ & \sum_{n=1}^{N} E_n \leq E_T. \end{aligned}$$

This problem can be seen as an application of water-filling theorem to multiple OFDM symbols as the sum rate is to be maximized with power constraint. The main difference is that we consider practical coding with a (coded) BER constraint. Thus, this formulation is generic and applicable to other problems where the power allocation is required to multiple codewords.

The solution to (6) can be found by an exhaustive search as follows. Firstly, we list all possible combinations of MCS candidates for N OFDM bands. (There are M^N possible combinations). Secondly, for each combination, where $\hat{m}_n$ is supposed to be used in OFDM band n, $n = 1, 2, \ldots, N$, find the set of power levels $\hat{E}_n$, $n = 1, 2, \ldots, N$, such that $\hat{m}_n = m_n^*(\hat{E}_n)$. If $\hat{m}_n > 1$, $\hat{E}_n$ is certainly unique since $P_b(E; \hat{m}_n, \hat{\mathbf{h}}_n)$ decreases with E. If this set of power levels satisfies the power budget constraint, this selection of MCS candidates and power levels conform a feasible solution. Lastly, among the feasible solutions, the optimal solution is determined by the one that can achieve the highest total data rate. However, since the complexity of an exhaustive search grows exponentially with N and becomes prohibitively high if N is relatively large, searching algorithms of lower complexity are preferred[2]. In the following, we will discuss two low complexity algorithms to derive good power allocation policies based on the greedy and dynamic programming principles.

B. Greedy algorithm

The algorithm based on the greedy principle is described as follows. The algorithm requires a number of steps. In the first step, all OFDM bands are assigned with $\eta^{(1)}$ (the first MCS candidate) and allocated by no transmission power. In each subsequent step, the MCS index for one of N OFDM bands is increased by 1. The OFDM band chosen to be reassigned is the one requiring the lowest amount of power increase for a unit of data rate increase while the BER constraint is still satisfied. Each step takes an amount of power from the power budget to allocate to the OFDM band that is reassigned with a new data rate. The algorithm is performed iteratively until the remaining power budget is not enough to increase the data rate in any

[2]In this paper, the complexity of an algorithm is roughly represented by the number of steps required to carry out the algorithm.

band. This algorithm is similar to the bit-loading technique found in the literature, e.g., [10] and references therein.

The advantage of the greedy algorithm is that its complexity is relatively low ($\mathcal{O}(MN)$). However, there are some issues open for discussion. First, the necessary condition for the greedy algorithm to be optimal is that for two different data rate levels, the higher one must require a larger amount of power to increase a unit of data rate than the other [11]. In practice, this condition may not be satisfied. For example, the channel codes in the second, third, and fourth MCS candidates in Table I have the diversity orders of 10, 5, and 6, respectively, Therefore, if the set of MCS candidates given in Table I is used, for some CIR realizations, increasing the data rate from 2 bps/Hz to 3 bps/Hz requires more additional amount of power than to increase from 3 bps/Hz to 4 bps/Hz. Due to the fact that the occurrence of this behavior is not often, the greedy algorithm can be seen as an effective algorithm provided that the power levels are unquantized, i.e., no constraint on the number of feedback bits used to feedback the power information. Second, if the number of these bits is limited, the amount of power taken from the power budget in each step should be in quantized levels. In this case, the greedy algorithm is no longer optimal. Therefore, we next develop another algorithm based on dynamic programming, which is optimal if only discrete levels of feedback power are considered.

C. Dynamic programming

Let Q, a positive integer, be the number of power steps. There are $Q+1$ power levels within the range from 0 to E_T can be used for allocation. We assume that these possible power levels are uniformly quantized. Therefore, $E_T = QE_0$, where E_0 is an identical changing step of power. Let $d_n, d_n \in \mathbb{Z}^+$, denote the multiple of E_0 allocated for the nth OFDM band, i.e., $E_n = d_n E_0$. In the power allocation problem, a set of integers $\{d_1, d_2, \ldots, d_N\}$ constrained by $\sum_{n=1}^{N} d_n \leq Q$ that maximizes the throughput are required to be found. Thus, the problem for the case of discrete power levels become

$$\begin{aligned} &\max_{\{d_n\}_{n=1}^{N}} \sum_{n=1}^{N} \eta_n^*(d_n E_0), \\ &\text{subject to } d_n \in \mathbb{Z}^+, \ \forall n : 1 \leq n \leq N; \\ &\sum_{n=1}^{N} d_n \leq Q, \end{aligned} \tag{7}$$

where $Q = E_T/E_0$ and $\eta_n^*(\cdot)$ is defined in (5).

To apply a dynamic programming technique to this optimization problem, the process of allocating power levels can be presented in a trellis that includes $N+1$ stages from 0 to N. At stage n, there are $Q+1$ possible states from 0 to Q, where each state represents the number of remaining unit powers that can be used to allocate for the OFDM bands from $n+1$ to N. Note that the initial state at $n = 0$ must be Q. From state q at stage n, the next state at stage $n+1$ cannot be greater than q. The output of a transition from q to q' is given by $f_n(q, q') = \eta_n^*((q - q')E_0)$, which is defined as the cost of a transition. Furthermore, the metric of a path through a sequence of states is defined by the sum of costs. Fig. 2 illustrates a graphical representation of the trellis. The optimization problem is to find the best path that has the maximum path metric. As the cost of a transition is based only on the starting and ending states, an algorithm based on dynamic programming can provide the optimal solution with a lower complexity. In particular, through the stages from 1 to N, a state at each stage keeps the previous state, which in fact stores the path that maximizes the sum of the costs up to the current state. The optimal path over all $N+1$ stages then can be found by tracing back from the state with highest path metric at stage N.

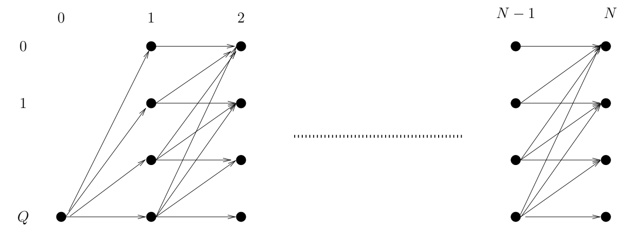

Fig. 2. Trellis presentation for power allocation.

TABLE I
LIST OF EMPLOYED MODULATION AND CODING SCHEMES.

MCS index	Modulation format	Code rate	Data rate (bps/Hz)
1			0
2	QPSK	1/2	1
3	16-QAM	1/2	2
4	16-QAM	3/4	3
5	64-QAM	2/3	4
6	64-QAM	3/4	4.5

Note that the complexity of this dynamic programming is $\mathcal{O}((Q+1)N)$, which could be higher than that of the greedy algorithm. However, with the discrete power constraint, i.e., $E_n \in \{0, E_0, , \ldots, QE_0\}$, the dynamic programming provides the optimal solution. Furthermore, this solution is an approximation for the original problem in (3) and this approximation can be improved by increasing Q. Through simulations in the next section, we can see that the dynamic programming is preferred for a relatively small value of Q.

IV. Simulation results

In all simulations, we choose $N = 8$ and $L = 1024$. We assume that CIR vectors are independent from one OFDM band to another. In each band, the CIR vector consists of $P = 128$ independent tap coefficients and has an identical exponential power delay profile, i.e., $h_{n,p} \sim \mathcal{CN}\left(0, \frac{\exp(-pT_s/T_{rms})}{1-\exp(-T_s/T_{rms})}\right), \forall n : 1 \leq n \leq N$, where T_s is the sampling period, and T_{rms} is the root mean square delay spread, which is assumed that $T_{rms} = 6T_s$.

For modulation and coding, the set of MCS candidates in Table I is used. For the MCS candidates with the code rate of half, the convolutional code of $(171, 133)$ octal generator is used, and it is defined as the mother code. For the MCS

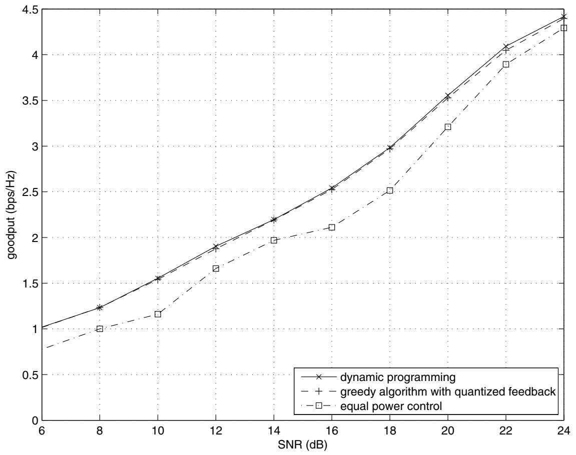

Fig. 3. System performances for different power control policies, $Q = 31$.

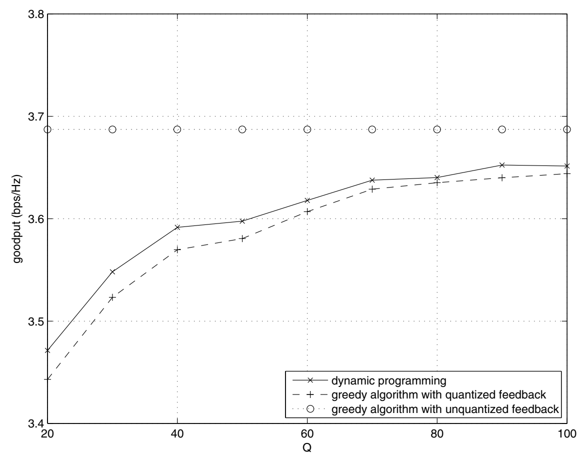

Fig. 4. Average goodput versus Q (the number of discrete power levels for power allocation), SNR = 20 dB.

candidates with code rates higher than half, the channel codes are obtained by puncturing from the mother code, where the puncturing patterns, the minimum Hamming distances, and the total input weights of error events for each Hamming distance are given in [12]. For BER estimation, i.e., $P_b(E; m, \mathbf{h})$, we use the derivation in [7].

For comparison purposes, we include in our simulations the symbol-level adaptive modulation and coding system with equal power control as proposed in [7]. We assume that the channel estimation and prediction are perfect. Furthermore, an error-free feedback link is assumed. The target BER is set by $\bar{P}_b = 10^{-4}$. To measure the performance, we only count the codewords that can be decoded without any bit error to compute the average successful bits per each subcarrier and refer to the measurement index as goodput. When $Q = 31$ is used, Fig. 3 illustrates the advantage of the system with our proposed power control method over the system with equal power control. For example, about 2 dB SNR gap at 2.5 bps/Hz goodput is exhibited. In our simulations, the greedy algorithm is performed in discrete power levels that are identical to that used by the dynamic programming. As shown in Fig. 3, the dynamic programming can provide a slightly better performance than the greedy algorithm.

To have a finer comparison between the dynamic programming and greedy algorithms, we carry out simulations for different values of Q at 20 dB OS-SNR. Fig. 4 shows that the dynamic programming always outperforms the greedy algorithm. However, the performance gap is relatively small. Therefore, for a high value of Q, i.e., the number of bits for the feedback of power levels is sufficiently large, the greedy algorithm could be a better choice due to its low complexity.

V. CONCLUDING REMARKS

In this paper, we proposed a power control method for multiband coded OFDM systems and facilitated the algorithms based on the greedy principle and dynamic programming. It can be seen that the proposed power control method is an application of water-filling theorem to multiple OFDM symbols, where each OFDM symbol is a codeword of possibly different code rate. Through the simulations, it was confirmed that the proposed method can increase the system throughput. The dynamic programming is optimal under the discrete power levels constraint. This algorithm is suitable for the case of a small number of feedback bits for power control, while the greedy algorithm is more promising due to its low complexity for the case of a sufficiently large number of feedback bits.

REFERENCES

[1] IEEE Standard for Information technology – Telecommunication and information exchange between systems – Local and metropolitan area networks – Specific requirements, *Part 11: Wireless LAN Medium Access Control (MAC) Physical Layer (PHY) Specifications*, June 2007.

[2] IEEE Standard for Local and metropolitan area networks, *Part 16: Air Interface for Broadband Wireless Access Systems*, May 2009.

[3] R. G. Gallager, *Information Theory and Reliable Communication*. John Wiley, 1968.

[4] K.-B. Song, A. Ekbal, S. T. Chung, and J. M. Cioffi, "Adaptive modulation and coding (AMC) for bit-interleaved coded OFDM (BIC-OFDM)," in *Proc. IEEE GLOBECOM '04*, vol. 6, pp. 3197–3201, 2004.

[5] H. Moon and D. C. Cox, "Efficient power allocation for coded OFDM systems," *IEEE Trans. Commun.*, vol. 57, pp. 943–947, Apr. 2009.

[6] A. Kenarsari-Anhari and L. Lampe, "Power allocation for coded OFDM via linear programming," *IEEE Commun. Lett.*, vol. 13, pp. 887–889, Dec. 2009.

[7] C. K. Sung, S.-Y. Chung, J. Heo, and I. Lee, "Adaptive bit-interleaved coded OFDM with reduced feedback information," *IEEE Trans. Commun.*, vol. 55, pp. 1649–1655, Sept. 2007.

[8] S. Nagaraj, "Symbol-level adaptive modulation for coded OFDM on block fading channels," *IEEE Trans. Commun.*, vol. 57, pp. 2872–2875, Oct. 2009.

[9] IEEE Standard for Information technology – Telecommunications and information exchange between systems – Local and metropolitan area networks – Specific requirements, *Part 15.3: Wireless Medium Access Control (MAC) and Physical Layer (PHY) Specifications for High Rate Wireless Personal Area Networks (WPANs)*, June 2003.

[10] N. Papandreou and T. Antonakopoulos, "Bit and power allocation in constrained multicarrier systems: the single-user case," *EURASIP J. Advances Signal Process.*, vol. 2008, Jan. 2008.

[11] A. Federgruen and H. Groenevelt, "The greedy procedure for resource allocation problems: Necessary and sufficient conditions for optimality," *Oper. Res.*, vol. 34, pp. 909–918, Nov.–Dec. 1986.

[12] P. Frenger, P. Orten, T. Ottosson, and A. Svensson, "Multi-rate convolutional codes," tech. rep., Chalmers University of Technology, 1998.